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RAREFIED CYLINDRICAL POISEUILLE GAS FLOW DUE TO HARMONICALLY OSCILLATING PRESSURE GRADIENT

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ABSTRACT

The pipe flow due to an externally imposed harmonically oscillating pressure gradient is a classical problem in fluid mechanics [1] and it is analytically solved by applying the unsteady Stokes equation subject to no-slip boundary conditions. However, as it is pointed in [2] this approach is valid under the provisions that both the mean free path and time are much smaller than the distance between the plates and the reference oscillating time respectively. If either of these restrictions is violated the problem must be tackled via kinetic theory.

In the present work, the oscillatory Poiseuille flow at arbitrary oscillation frequency over the whole range of the Knudsen number is investigated. Assuming that the amplitude of the oscillation is small the problem is modelled by the linearized time-dependent Bhatnagar–Gross–Krook (BGK) kinetic equation subject to diffuse boundary conditions. This type of flow remains an active area of research due to “Richardson annular effect” [1] in which the velocity exhibits its maximum within the viscous Stokes wall layer and not, as expected, in the center of the velocity field. It appears in high frequencies and it causes anomalous solid formation in low-pressure chemical vapor deposition [3].

Consider the isothermal flow of a monoatomic rarefied through an infinite long circular tube of radius R . The gas is oscillating due to an imposed harmonically oscillating pressure gradient $d\tilde{P}(z',t')/dz' = \Re[\exp(-i\omega t')dP(z')/dz']$, where \Re denotes the real part of the complex expression $i = \sqrt{-1}$, t' is the time independent variable, $dP(z')/dz'$ is the amplitude of the oscillating pressure gradient and ω is the oscillation frequency, caused by a periodically moving membrane or piston. The flow is assumed to be harmonic in time, fully developed (independent of z) and varying in the r -direction. This oscillatory tube flow is characterized by two parameters: i) the rarefaction parameter $\delta = (P_0 R)/(\mu_0 \nu_0)$, where P_0 is a reference pressure, μ_0 is the gas viscosity at reference temperature T_0 and ν_0 is the most probable molecular velocity; ii) the oscillation speed parameter $\theta = P_0/(\mu_0 \omega)$. The flow is in the hydrodynamic regime provided that both $\delta \gg 1$ and $\theta \gg 1$.

To obtain a valid solution in the whole range of the gas rarefaction and for any oscillating frequency a kinetic theory approach is applied [2]. Furthermore, assuming that the amplitude of the oscillation is adequately small, linearization is introduced. The main unknown is a complex perturbed distribution function of the form $\tilde{Y}(t', r, c_r, c_\varphi) = \Re[Y(r', c_r, c_\varphi) \exp(-i\omega t')]$ where $c_r = \zeta \cos \varphi$ and $c_\varphi = \zeta \sin \varphi$ are the molecular velocities defined by their magnitude $\zeta \in [0, \infty]$ and polar angle

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$\varphi \in [0, 2\pi]$. Substituting this expression into the unsteady linearized BGK kinetic equation and following some routine manipulation it is deduced that $Y(r, c_r, c_\varphi)$ obeys the complex integro-differential equation

$$\zeta \cos \varphi \frac{\partial Y}{\partial r} - \frac{\zeta \sin \varphi}{r} \frac{\partial Y}{\partial \varphi} + \left(\delta - i \frac{\delta}{\theta} \right) Y = \delta u - \frac{1}{2}, \quad (1)$$

where $0 \leq r \leq 1$ and $u(r) = (1/\pi) \iint \zeta Y(r, c_r, c_\varphi) e^{-\zeta^2} d\zeta d\varphi$ is the bulk velocity. Since u is complex it may be written as $u = u_A(r) \exp[iu_p(r)]$, where u_A and u_p are the amplitude and the phase of the bulk velocity. The associated boundary conditions are $Y(1, \zeta, \varphi) = 0$ for $\pi/2 < \varphi < 3\pi/2$ and $Y(0, \zeta, \varphi) = Y(0, \zeta, \varphi - \pi)$ for $0 < \varphi < \pi/2$ and $3\pi/2 < \varphi < 2\pi$. In this flow configuration the main quantity of practical interest is the oscillatory complex, flow rate, denoted as $G = G_A \exp[iG_p]$.

In Figure 1, the flow rate amplitude is presented over δ/θ . It is clearly seen that for large values of θ the amplitude flow rate depends non-monotonically on δ/θ . For $\theta = 0.1, 1$ it is decreased as δ is increased due to high oscillation frequency. In addition, velocity amplitude is presented over r at $\delta = 1$ and $\theta = 0.1, 1, 10$. Richardson effect is observed at $\theta = 0.1$ where the oscillation frequency is high enough to increase the velocity higher near the wall than in the center. There, the viscous stresses created at the wall are slightly displaced from the wall due to viscous diffusion. However they are still strong enough to aid the pressure force, which is quickly reversed due to high frequency. The combination of these forces accelerates the fluid to produce the overshoot. In the center of the tube velocity is almost constant indicating that the flow is dominated by inertia. In the other two cases the usual velocity parabolic profile is established.

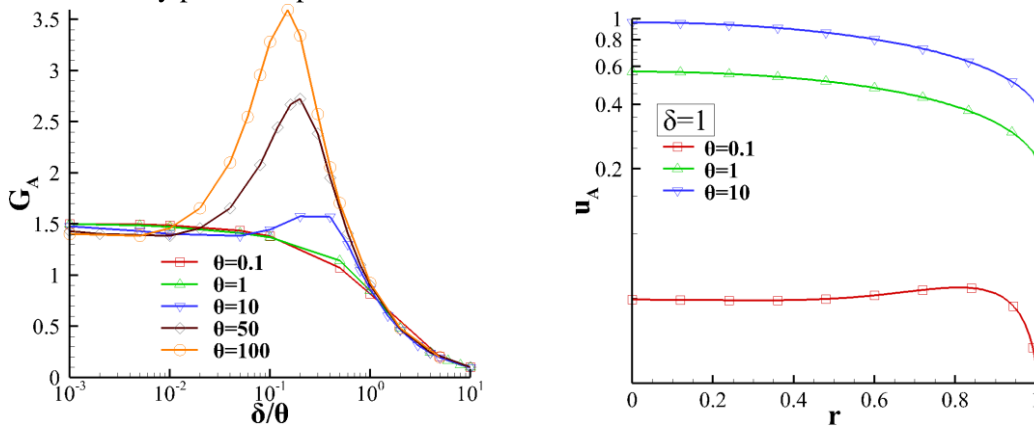


Figure 1: Flow rate amplitude versus δ/θ and velocity amplitude versus r at $\delta = 1$ and $\theta = 0.1, 1, 10$

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